2.1

T9

$y = x^2 - 2x - 3 \quad P(2,-3)$

求P点处的斜率及切线方程

$$\frac{\Delta y}{\Delta h} = \frac{(2+h)^2 - 2(2+h) - 3 - (2^2 - 2\times 2 - 3)}{h}$$

$$= \frac{h^2+2h}{h} = h+2$$

$$\lim_{h\to 0} \frac{\Delta y}{\Delta h} = \lim_{h\to 0} h+2 = 2.$$

因此斜率是2.

$$\begin{cases} y = 2x+b \\ x = 2 \\ y = -3 \end{cases} \Rightarrow b = -7$$

切线方程为 $y = 2x - 7$

2.2

T3

T T F F F T T

T22

$$\lim_{h\to 0} \frac{\sqrt{5h+4} - 2}{h}$$

$$= \lim_{h\to 0} \frac{(\sqrt{5h+4} - 2)(\sqrt{5h+4} + 2)}{h(\sqrt{5h+4} + 2)}$$

$$= \lim_{h\to 0} \frac{5h}{h(\sqrt{5h+4} + 2)}$$

$$= \lim_{h\to 0} \frac{5}{\sqrt{5h+4} + 2} = \frac{5}{4}$$

T42

$$\lim_{x\to 4} \frac{4-x}{5 - \sqrt{x^2+9}}$$

$$= \lim_{x\to 4} \frac{(4-x)(5+\sqrt{x^2+9})}{(5-\sqrt{x^2+9})(5+\sqrt{x^2+9})}$$

$$= \lim_{x\to 4} \frac{(4-x)(5+\sqrt{x^2+9})}{16 - x^2}$$

$$= \lim_{x\to 4} \frac{(4-x)(5+\sqrt{x^2+9})}{(4+x)(4-x)}$$

$$= \lim_{x\to 4} \frac{5+\sqrt{x^2+9}}{4+x} = 5/4$$

T47

$$\lim_{x\to 0} \frac{1+x+\sin x}{3\cos x}$$

$$= \frac{\lim_{x\to 0} 1+x+\sin x}{\lim_{x\to 0} 3\cos x} = 1/3$$

T63

Q: If $\sqrt{5-2x^2} \le f(x) \le \sqrt{5-x^2}$ for $-1 \le x \le 1$, find $\lim_{x\to 0} f(x)$

A:

$$\lim_{x\to 0} \sqrt{5-2x^2} = \sqrt{5}$$

$$\lim_{x\to 0} \sqrt{5-x^2} = \sqrt{5}$$

由夹逼定理, 得

$$\lim_{x\to 0} f(x) = \sqrt{5}.$$

T78

Q: If $\lim_{x\to -2} \frac{f(x)}{x^2} = 1$

find (a) $\lim_{x\to -2} f(x)$

(b) $\lim_{x\to -2} \frac{f(x)}{x}$

(a)

$$\lim_{x\to -2} f(x) = \lim_{x\to -2} \frac{f(x)}{x^2} x^2$$

$$= \lim_{x\to -2} \frac{f(x)}{x^2} \lim_{x\to -2} x^2$$

$$= 1 \cdot 4 = 4$$

(b)

$$\lim_{x\to -2} \frac{f(x)}{x}$$

$$= \lim_{x\to -2} \frac{f(x)}{x^2} \cdot x$$

$$= \lim_{x\to -2} \frac{f(x)}{x^2} \lim_{x\to -2} x$$

$$= 1 \cdot (-2) = -2$$

2.3

T49 Q: 证明 $\lim_{x\to 0} x\sin\frac{1}{x} = 0$

A: $-|x| \leq x\sin\frac{1}{x} \leq |x|$

$\lim_{x\to 0}|x| = \lim_{x\to 0} -|x| = 0$

2.4

T1

a. T　b. T

c. F　d. T

e. T　f. T

g. F　h. F

i. F　j. F

k. T　l. F

T5.

a. No. $\sin\frac{1}{x}$ 振荡并不趋于任何一个值.

b. Yes. $\lim_{x\to 0^-} f(x) = 0$

c. No. 因为 $\lim_{x\to 0^+} f(x)$ 不存在.

T15.

$$\lim_{h\to 0^+}\frac{\sqrt{h^2+4h+5}-\sqrt{5}}{h}$$

$$= \lim_{h\to 0^+}\frac{(\sqrt{h^2+4h+5}-\sqrt{5})(\sqrt{h^2+4h+5}+\sqrt{5})}{h(\sqrt{h^2+4h+5}+\sqrt{5})}$$

$$= \lim_{h\to 0^+}\frac{h^2+4h}{h(\sqrt{h^2+4h+5}+\sqrt{5})} = \lim_{h\to 0^+}\frac{h+4}{\sqrt{h^2+4h+5}+\sqrt{5}} = \frac{2}{\sqrt{5}}$$

T19

$$\lim_{\theta\to 3^+}\frac{\lfloor\theta\rfloor}{\theta} = \lim_{\theta\to 3^+}\frac{3}{\theta} = \frac{3}{3} = 1$$

T34

$$\lim_{h\to 0}\frac{\sin(\sinh)}{\sinh}$$

令 $\sinh = \theta$

$$= \lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1$$

T41

$$\lim_{\theta\to 0}\frac{\tan\theta}{\theta^2\cot 3\theta}$$

$$= \lim_{\theta\to 0}\frac{\sin\theta/\cos\theta}{\theta^2\,\frac{\cos 3\theta}{\sin 3\theta}}$$

$$= \lim_{\theta\to 0}\frac{\sin\theta}{\theta}\,\frac{3\sin 3\theta}{3\theta}\,\frac{1}{\cos\theta\cdot\cos 3\theta}$$

$$= 1\cdot 3\cdot\frac{1}{1\cdot 1} = 3$$

T45

Q: f is an odd function. we've known $\lim_{x\to 0^+} f(x) = 3$, find $\lim_{x\to 0^-} f(x)$.

$$\lim_{x\to 0^-} f(x) \overset{令 t=-x}{=\!=\!=} \lim_{t\to 0^+} f(-t)$$

$$= \lim_{t\to 0^+} -f(t)$$

$$= -\lim_{t\to 0^+} f(t)$$

$$= -3$$

补充作业(week 1)

CH2

T1

$\lim\limits_{x\to 0^-} x-\sin x=0$ 且 $x<0$ 时 $x-\sin x<0$.

令 $t=x-\sin x$. 当 $x\to 0^-$, $t\to 0^-$.

$\lim\limits_{x\to 0^-} f(x-\sin x)=\lim\limits_{t\to 0^-} f(t)=b$.

同理 $\lim\limits_{x\to 0^-} f(x^2+x)=b$

选择 D. 3b

T2

(2)

$$\lim_{x\to 0}\frac{\sin(1-\cos x)}{(\tan x)^2}$$

$$=\lim_{x\to 0}\frac{\sin(1-\cos x)\cos^2 x}{1-\cos^2 x}$$

$$\overset{令t=1-\cos x}{=}\lim_{t\to 0}\frac{\sin t\,(1-t)^2}{(2-t)t}$$

$$=\lim_{t\to 0}\frac{\sin t}{t}\cdot\frac{(1-t)^2}{2-t}=1\cdot\frac{1}{2}=1/2$$

(12)

$$\frac{1}{x}-1<\left\lfloor\frac{1}{x}\right\rfloor\leq\frac{1}{x}$$

$$\lim_{x\to 0}\left(\frac{1}{x}-1\right)\sin x=\lim_{x\to 0}\frac{\sin x-x\sin x}{x}$$

$$=\lim_{x\to 0}\frac{\sin x}{x}-\lim_{x\to 0}\sin x$$

$$=1-0=1$$

$$\lim_{x\to 0}\frac{1}{x}\sin x=1$$

故 $\lim\limits_{x\to 0}\left\lfloor\frac{1}{x}\right\rfloor\sin x=1$.

T6.

$$\lim_{x\to\pi}\frac{\sin(mx)}{\sin(nx)}$$

$$=\lim_{x\to\pi}\frac{\sin(m(x-\pi)+m\pi)}{\sin(n(x-\pi)+n\pi)}$$

$$=\lim_{t\to 0}\frac{\sin(mt+m\pi)}{\sin(nt+n\pi)}$$

(1) m奇, n奇

$$原式=\lim_{t\to 0}\frac{-\sin(mt)}{-\sin(nt)}$$

$$=\lim_{t\to 0}\frac{\sin(mt)}{mt}\,\frac{mt}{nt}\,\frac{nt}{\sin(nt)}$$

$$=1\cdot\frac{m}{n}\cdot 1=\frac{m}{n}.$$

(2) m奇, n偶

$$原式=\lim_{t\to 0}-\frac{\sin(mt)}{\sin(nt)}=-\frac{m}{n}$$

(3) m偶, n奇

$$原式=\lim_{t\to 0}\frac{\sin(mt)}{-\sin(nt)}=-\frac{m}{n}$$

(4) m偶, n偶

$$原式=\lim_{t\to 0}\frac{\sin(mt)}{\sin(nt)}=\frac{m}{n}$$

T10

Q: Find a, b s.t.

$$\lim_{x\to\pi/2}\frac{\sqrt{x}-a}{\cos x}=b$$

为使极限存在

$$\lim_{x\to\pi/2}\sqrt{x}-a=0.$$

故 $a=\lim\limits_{x\to\pi/2}\sqrt{x}=\sqrt{\pi/2}$.

$$b=\lim_{x\to\pi/2}\frac{\sqrt{x}-\sqrt{\pi/2}}{\cos x}$$

$$\lim_{x\to\pi/2}\frac{\sqrt{x}-\sqrt{\pi/2}}{\cos x}$$

$$=\lim_{x\to\pi/2}\frac{(\sqrt{x}-\sqrt{\pi/2})(\sqrt{x}+\sqrt{\pi/2})}{\cos x(\sqrt{x}+\sqrt{\pi/2})}$$

$$=\lim_{x\to\pi/2}\frac{x-\pi/2}{\cos x(\sqrt{x}+\sqrt{\pi/2})}$$

$$=\lim_{x\to\pi/2}\frac{x-\pi/2}{\cos x}\lim_{x\to\pi/2}\frac{1}{\sqrt{x}+\sqrt{\pi/2}}$$

对第一个极限 令 $x-\pi/2=t$

$$=\lim_{t\to 0}\frac{t}{\cos(t+\pi/2)}\quad\frac{1}{2\sqrt{\pi/2}}$$

$$=\lim_{t\to 0}-\frac{t}{\sin t}\quad\frac{1}{2\sqrt{\pi/2}}$$

$$=-\frac{1}{2\sqrt{\pi/2}}$$